Hoofdstuk 8 Goniometrie

8.1 De eenheidscirkel

Opgave 1:

a. $\sin 65° = \frac{PQ}{1}$

$PQ = \sin 65° = 0{,}91$

$\cos 65° = \frac{OQ}{1}$

$OQ = \cos 65° = 0{,}42$

b. $P(0{,}42\,;0{,}91)$

Opgave 2:

a. $\angle POQ = 180° - 115° = 65°$

$PQ = 0{,}91$

$OQ = 0{,}42$

$P(-0{,}42\,;0{,}91)$

b. $\cos 115° = -0{,}42$

$\sin 115° = 0{,}91$

$x_P = \cos 115° = -0{,}42$

$y_P = \sin 115° = 0{,}91$

c. $\angle POQ = 180° + 65° = 245°$

$P(-0{,}42\,;-0{,}91)$

$x_P = \cos 245° = -0{,}42$

$y_P = \sin 245° = -0{,}91$

Opgave 3:

a. $\sin 0° = 0$

b. $\cos °0° = 1$

c. $\sin 90° = 1$

d. $\cos 90° = 0$

e. $\sin 270° = -1$

f. $\cos 270° = 0$

g. $\sin 360° = 0$

h. $\cos 360° = 1$

i. $\sin 450° = 1$

j. $\cos(-90°) = 0$

k. $\sin(-540°) = 0$

l. $\cos 1080° = 1$

m. $\sin 1980° = 0$

n. $\cos(-180°) = -1$

o. $\sin 990° = -1$

Opgave 4:

$P(\cos 110°, \sin 110°) = (-0{,}34; 0{,}94)$

$Q(\cos 200°, \sin 200°) = (-0{,}94; -0{,}34)$

$R(\cos(-102°), \sin(-102°)) = (-0{,}21; -0{,}98)$

$S(\cos(-50°), \sin(-50°)) = (0{,}64; -0{,}77)$

Opgave 5:

$\frac{360°}{5} = 72°$

$B(2\cos 72°, 2\sin 72°) = (0{,}62; 1{,}90)$

$C(2\cos 144°, 2\sin 144°) = (-1{,}62; 1{,}18)$

$D(2\cos 216°, 2\sin 216°) = (-1{,}62; -1{,}18)$

$E(2\cos 288°, 2\sin 288°) = (0{,}62; -1{,}90)$

Opgave 6:

a. $\frac{360°}{8} = 45°$

$P(\cos 45°, \sin 45°) = (0{,}71; 0{,}71)$

b. $2 \cdot 45° = 90°$

$P(\cos 90°, \sin 90°) = (0{,}1)$

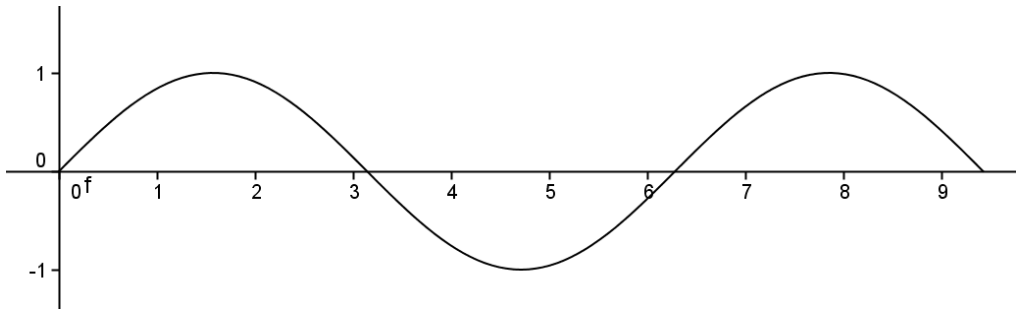
c. $3\frac{1}{2} \cdot 45° = 157{,}5°$

$P(\cos 157{,}5°, \sin 157{,}5°) = (-0{,}92; 0{,}38)$

d.

t	0	1	2	3	4	5	6	7	8	9	10	11	12
y_P	0	0,7	1	0,7	0	−0,7	−1	−0,7	0	0,7	1	0,7	0

e.



Opgave 7:

a. $\cos\alpha = 0{,}81$

$\alpha = 36°$

b. $\sin\alpha = 0{,}94$

$\alpha = 70°$ op GR

dus $\alpha = 110°$

c. $\cos\alpha = 0{,}26$

$\alpha = 75°$ op GR

dus $\alpha = -75°$

d. $\sin\alpha = -0{,}22$

$\alpha = -13°$ op GR

dus $\alpha = 193°$

Opgave 8:

$y_P = 0{,}92$

$\sin \angle P = 0{,}92$

$\angle P = 66{,}9°$

$x_Q = -0{,}87$

$\cos \angle Q = -0{,}87$

$\angle Q = 150{,}5°$ op GR

$\angle Q = -150{,}5° + 360° = 209{,}5°$

$\angle POQ = 209{,}5° - 66{,}9° = 143°$

8.2 Radialen

Opgave 9:

a. $omtrek = 2\pi\, r = 2\pi \cdot 1 = 2\pi$

b. bij $90°$ heb je $\frac{1}{4}$ deel van de cirkel doorlopen, dus de cirkelboog is $\frac{1}{4} \cdot 2\pi = \frac{1}{2}\pi$

c.

draaiingshoek α	$0°$	$90°$	$180°$	$270°$	$360°$
Lengte cirkelboog b	0	$\frac{1}{2}\pi$	π	$1\frac{1}{2}\pi$	2π

Opgave 10:

a. $P(\cos 5, \sin 5) = (0{,}28; -0{,}96)$

b. $P(\cos 6, \sin 6) = (0{,}96; -0{,}28)$

c. $P(\cos 20, \sin 20) = (0{,}41; 0{,}91)$

Opgave 11:

a. $P(\cos \frac{1}{2}\pi, \sin \frac{1}{2}\pi) = (0, 1)$

b. $P(\cos \pi, \sin \pi) = (-1, 0)$

c. $P(\cos 1\frac{1}{2}\pi, \sin 1\frac{1}{2}\pi) = (0, -1)$

Opgave 12:

a. $\frac{1}{6}\pi\ rad = 30°$

b. $\frac{1}{4}\pi\ rad = 45°$

c. $2\pi\ rad = 360°$

d. $2\ rad = \frac{360}{\pi}° = 114{,}6°$

e. $\frac{5}{4}\pi\ rad = 225°$

f. $\frac{5}{4}\ rad = \frac{225°}{\pi} = 71{,}6°$

g. $-2\frac{1}{3}\pi\ rad = -420°$

h. $-2\frac{1}{3}\ rad = \frac{-420°}{\pi} = -133{,}7°$

Opgave 13:

a. $360° = 2\pi\ rad$

b. $30° = \frac{1}{6}\pi\ rad$

c. $-45° = -\frac{1}{4}\pi\ rad$

d. $60° = \frac{1}{3}\pi\ rad$

e. $90° = \frac{1}{2}\pi\ rad$

f. $135° = \frac{3}{4}\pi\ rad$

g. $-75° = -\frac{75}{180}\pi\ rad = -\frac{5}{12}\pi\ rad$

h. $240° = 1\frac{1}{3}\pi\ rad$

i. $300° = 1\frac{2}{3}\pi\ rad$

j. $720° = 4\pi\ rad$

k. $400° = \frac{400}{180}\pi\ rad = 2\frac{2}{9}\pi\ rad$

l. $0° = 0\ rad$

m. $210° = 1\frac{1}{6}\pi\ rad$

n. $5° = -\frac{5}{180}\pi\ rad = -\frac{1}{36}\pi\ rad$

o. $540° = 3\pi\ rad$

p. $1° = \frac{1}{180}\pi\ rad$

Opgave 14:

a. $7° = \frac{7}{360} \cdot 2\pi = 0{,}12\ rad$

b. $18° = \frac{18}{360} \cdot 2\pi = 0{,}31\ rad$

c. $-51{,}3° = \frac{-51{,}3}{360} \cdot 2\pi = -0{,}90\ rad$

d. $1{,}7° = \frac{1{,}7}{360} \cdot 2\pi = 0{,}03\ rad$

e. $-320° = \frac{-320}{360} \cdot 2\pi = -5{,}59\ rad$

f. $1030° = \frac{1030}{360} \cdot 2\pi = 17{,}98\ rad$

g. $90° = \frac{90}{360} \cdot 2\pi = 1{,}57\ rad$

h. $57° = \frac{57}{360} \cdot 2\pi = 0{,}99\ rad$

Opgave 15:

hoek in graden	0°	30°	45°	60°	90°	135°	180°	240°	315	360°
hoek in radialen	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$1\frac{1}{3}\pi$	$1\frac{3}{4}\pi$	2π

Opgave 16:

a. $\cos\frac{2}{3}\pi = -0{,}50$

b. $\cos\frac{2}{3} = 0{,}79$

c. $\sin\frac{4}{5}\pi = 0{,}59$

d. $\sin\frac{4}{5} = 0{,}72$

e. $\cos 7{,}6\pi = 0{,}31$

f. $\cos 7{,}6 = 0{,}25$

Opgave 17:

a. $\sin\alpha = 0{,}92$
   $\alpha = 1{,}17$

b. $\cos\alpha = 0{,}85$
   $\alpha = 0{,}55$

c. $\sin\alpha = \frac{5}{12}$
   $\alpha = 0{,}43$

d. $\cos\alpha = \frac{3}{17}$
   $\alpha = 1{,}39$

e. $\sin\alpha = \frac{1}{3}\sqrt{5}$
   $\alpha = 0{,}84$

f. $\cos\alpha = \frac{1}{4}\sqrt{2}$
   $\alpha = 1{,}21$

Opgave 18:

a. $\sin\alpha = 0{,}35$

$\alpha = 0{,}36$ op de GR

$\alpha = \pi - 0{,}36 = 2{,}78$

b. $\cos\alpha = -0{,}35$

$\alpha = 1{,}93$ op de GR

$\alpha = -1{,}93 + 2\pi = 4{,}35$

Opgave 19:

$\cos\angle P = -0{,}32$

$\angle P = 1{,}897$

$\sin\angle Q = -0{,}88$

$\angle Q = -1{,}076$ op de GR

$\angle Q = \pi + 1{,}076 = 4{,}217$

$\angle POQ = 4{,}217 - 1{,}897 = 2{,}32$

Opgave 20:

a. punt P zit $23 - 15 - 1 = 7$ m boven de as

$\sin\alpha = \frac{7}{15}$

$\alpha = 0{,}486$

draaiingshoek $= \frac{1}{2}\pi + 0{,}486 = 2{,}06$

b. draaiingshoek $= \frac{1}{2}\pi + \pi - 0{,}486 = 4{,}23$

Opgave 21:

a. *

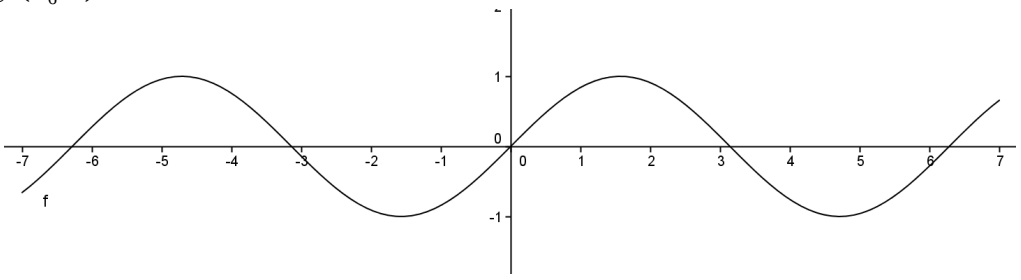
b. $f(\frac{1}{6}\pi) = 0{,}5$

$f(\frac{5}{6}\pi) = 0{,}5$

$f(1\frac{1}{6}\pi) = -0{,}5$

$f(1\frac{5}{6}\pi) = -0{,}5$

c.



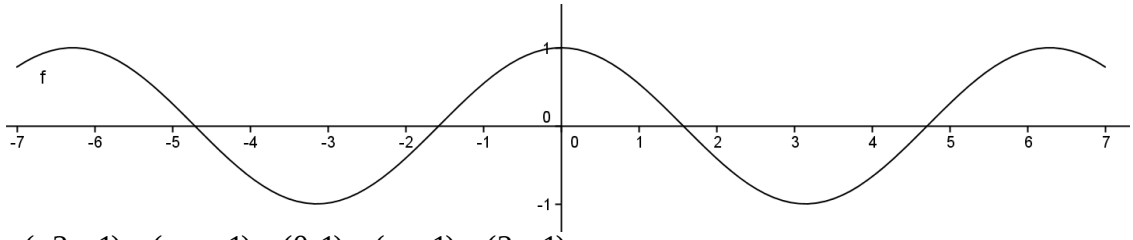
toppen: $(-1\frac{1}{2}\pi, 1)$ $(-\frac{1}{2}\pi, -1)$ $(\frac{1}{2}\pi, 1)$ $(1\frac{1}{2}\pi, -1)$

d. $(-2\pi, 0)$ $(-\pi, 0)$ $(0, 0)$ $(\pi, 0)$ $(2\pi, 0)$

Opgave 22:

a. *

b.

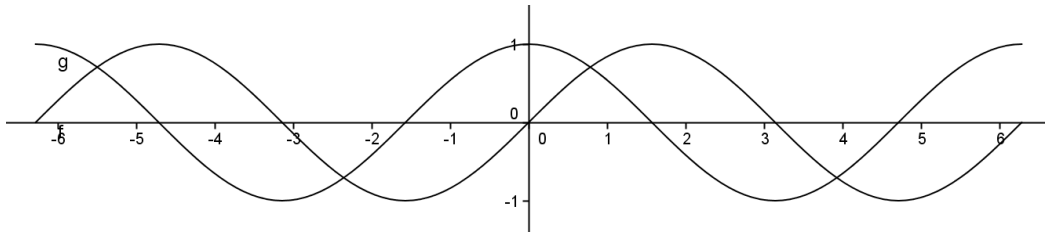


c. $(-2\pi,1)$ $(-\pi,-1)$ $(0,1)$ $(\pi,-1)$ $(2\pi,1)$

d. $(-1\frac{1}{2}\pi,0)$ $(-\frac{1}{2}\pi,0)$ $(\frac{1}{2}\pi,0)$ $(1\frac{1}{2}\pi,0)$

Opgave 23:

a.



b. $\frac{1}{4}\pi\ rad = 45°$ dus je hebt in de eenheidscirkel te maken met een geodriehoek, waardoor de horizontale rechtshoekszijde even lang is als de verticale rechthoekszijde.

$x = -1\frac{3}{4}\pi \quad \vee \quad x = -\frac{3}{4}\pi \quad \vee \quad x = \frac{1}{4}\pi \quad \vee \quad x = 1\frac{1}{4}\pi$

8.3 Transformaties en sinusoïden

Opgave 24:

a. $T(0,2)$
ev.as: 2

b. $T(3,0)$

c. $V_{x-as,4}$
amp: 4

d. $V_{y-as,\frac{1}{5}}$
per $= \frac{2\pi}{5} = \frac{2}{5}\pi$

Opgave 25:

a. 1. $V_{x-as,2}$
2. $T(-3,0)$
ev.as: 0
amp: 2
per: 2π
bp: $(-3,0)$

b. 1. $V_{x-as,\frac{1}{3}}$
2. $T(0,\frac{1}{5})$
ev.as: $\frac{1}{5}$
amp: $\frac{1}{3}$
per: 2π
bp: $(0,\frac{1}{5})$

c. 1. $V_{y-as,\frac{1}{3}}$
2. $T(4,0)$
ev.as: 0
amp: 1
per: $\frac{2\pi}{3} = \frac{2}{3}\pi$
bp: (4,1)

d. 1. $V_{x-as,1\frac{1}{2}}$
2. $V_{y-as,4}$
ev.as: 0
amp: $1\frac{1}{2}$
per: $\frac{2\pi}{\frac{1}{4}} = 8\pi$
bp: $(0,1\frac{1}{2})$

Opgave 26:

a. 1. $V_{x-as,1.2}$
2. $T(\frac{1}{6}\pi,5)$
ev.as: 5
amp: 1.2

per: 2π
bp: $(\frac{1}{6}\pi, 6.2)$

b. 1. $V_{y-as,5}$
2. $T(-\frac{1}{3}\pi, 0.4)$
ev.as: 0,4
amp: 1
per: $\frac{2\pi}{\frac{1}{5}} = 10\pi$
bp: $(-\frac{1}{3}\pi, 0.4)$

c. 1. $V_{x-as,0.29}$
2. $V_{y-as,\frac{1}{3}}$
3. $T(-1.4, 0)$
ev.as: 0
amp: 0,29
per: $\frac{2\pi}{3} = \frac{2}{3}\pi$
bp: $(-1.4, 0.29)$

d. 1. $V_{x-as,2}$
2. $V_{y-as,\frac{1}{3}}$
3. $T(\frac{1}{2}\pi, -0.8)$
ev.as: $-0{,}8$
amp: 2
per: $\frac{2\pi}{3} = \frac{2}{3}\pi$
bp: $(\frac{1}{2}\pi, -0.8)$

Opgave 27:

$y = \sin x \xrightarrow{V_{y-as,3}} y = \sin \frac{1}{3}x \xrightarrow{T(4,-1.5)} y = -1{,}5 + \sin \frac{1}{3}(x-4)$

Opgave 28:

a. $y = \cos x \xrightarrow{T(\frac{1}{4}\pi, 4)} y = 4 + \cos(x - \frac{1}{4}\pi) \xrightarrow{V_{x-as,3}} y = 12 + 3\cos(x - \frac{1}{4}\pi)$

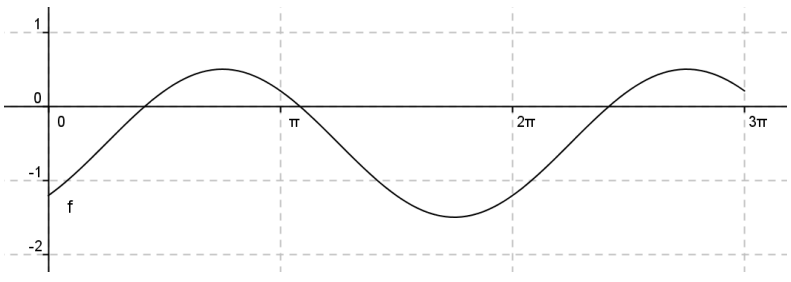
b. $y = \cos x \xrightarrow{V_{x-as,3}} y = 3\cos x \xrightarrow{T(\frac{1}{4}\pi, 4)} y = 4 + 3\cos(x - \frac{1}{4}\pi)$

Opgave 29:

De grafiek van f hoort bij figuur b.
De grafiek van g hoort bij figuur c.
De grafiek van h hoort bij figuur d.
De grafiek van k hoort bij figuur a.

Opgave 30:

a. ev.as: $-\frac{1}{2}$
amp: 1
per: 2π
bp: $(\frac{1}{4}\pi, -\frac{1}{2})$



b. $(\frac{1}{4}\pi,-\frac{1}{2})$ $(1\frac{1}{4}\pi,-\frac{1}{2})$ $(2\frac{1}{4}\pi,-\frac{1}{2})$

c. $(\frac{3}{4}\pi,\frac{1}{2})$ $(1\frac{3}{4}\pi,-1\frac{1}{2})$ $(2\frac{3}{4}\pi,\frac{1}{2})$

d. $x_C - x_A = 2\pi$ (precies de periode)

Opgave 31:

a. 1. $V_{x-as,3}$

2. $T(\frac{1}{4}\pi,2)$

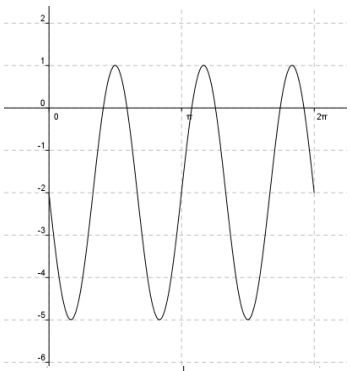
ev.as: 2
amp: 3
per: 2π
bp: $(\frac{1}{4}\pi,2)$

b. ev.as: 4
amp: 2
per: 2π
bp: $(\frac{1}{3}\pi,4)$

c. ev.as: 0
amp: 1
per: $\frac{2\pi}{3} = \frac{2}{3}\pi$
bp: $(\frac{1}{2}\pi,0)$

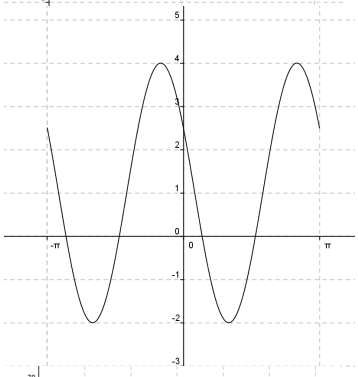
Opgave 32:

$f(x) = -2 + 3\sin 3(x + \frac{1}{3}\pi)$
ev.as: -2
amp: 3
per: $\frac{2\pi}{3} = \frac{2}{3}\pi$
bp: $(-\frac{1}{3}\pi,-2)$



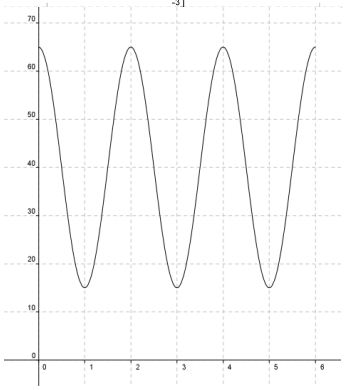
Opgave 33:

$f(x) = 1 + 3\cos 2(x + \frac{1}{6}\pi)$
ev.as: 1
amp: 3
per: $\frac{2\pi}{2} = \pi$
bp: $(-\frac{1}{6}\pi,4$



Opgave 34:

a. ev.as: 40
amp: 25
per: $\frac{2\pi}{\pi} = 2$
bp: $(1\frac{1}{2},40)$



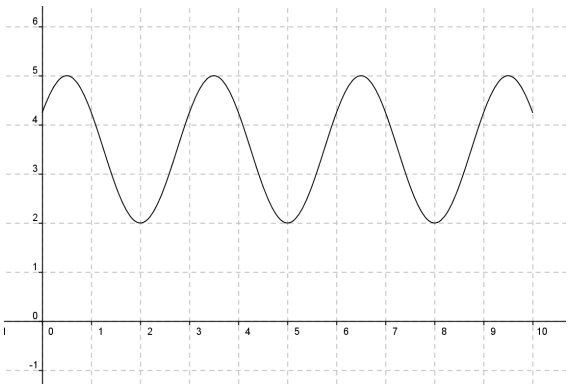
b. $y_1 = 40 + 25\sin\pi(x - 1\frac{1}{2})$ en $y_2 = 30$

intersect geeft:

$x = 0{,}63 \quad \vee \quad x = 1{,}37 \quad \vee \quad x = 2{,}63 \quad \vee \quad x = 3{,}37 \quad \vee \quad x = 4{,}63 \quad \vee \quad x = 5{,}37$

dus $0{,}63 < t < 1{,}37 \quad \vee \quad 2{,}63 < t < 3{,}37 \quad \vee \quad 4{,}63 < t < 5{,}37$

c. $\left[\frac{dy}{dx}\right]_{x=1\frac{1}{2}} = 78{,}5$



Opgave 35:

a. ev.as: 3,5

amp: 1,5

per: $\frac{2\pi}{\frac{2}{3}\pi} = 3$

bp: (0.5,5)

b. $y_1 = 1{,}5\cos(\frac{2}{3}\pi(x - 0{,}5)) + 3{,}5$

$y_2 = 4$

intersect geeft:

$x = 1{,}09 \quad \vee \quad x = 2{,}91 \quad \vee \quad x = 4{,}09 \quad \vee \quad x = 5{,}91 \quad \vee \quad x = 7{,}09 \quad \vee \quad x = 8{,}91$

dus: $0 \le t < 1{,}09 \quad \vee \quad 2{,}91 < t < 4{,}09 \quad \vee \quad 5{,}91 < t < 7{,}09 \quad \vee \quad 8{,}91 < t \le 10$

c. $\left[\frac{dy}{dx}\right]_{x=0} = 2{,}72$

d. snijpunt ev.as: $t = 0{,}5 + \frac{3}{4} \cdot 4 = 2{,}75$

$\left[\frac{dy}{dx}\right]_{x=2{,}75} = 3{,}14$

Opgave 36:

a. ev.as: 3

amp: 2

per: 2π

b. bp: $x = \frac{1}{3}\pi \quad y = 2$

$f(x) = 3 + 2\sin(x - \frac{1}{3}\pi)$

Opgave 37:

a. ev.as: $\frac{60 + -20}{2} = 20$

amp: $60 - 20 = 40$

per =50 dus $c = \frac{2\pi}{50} = \frac{1}{25}\pi$

bp: (0,20)

$y = 20 + 40\sin\frac{1}{25}\pi x$

b. bp: $(12\frac{1}{2}, 60)$ dus $y = 20 + 40\cos\frac{1}{25}\pi(t - 12\frac{1}{2})$

Opgave 38:

a. ev.as: $\frac{-220 + 100}{2} = -60$

amp: $100 - -60 = 160$

per: 6,8 dus $c = \frac{2\pi}{6{,}8} = \frac{\pi}{3{,}4}$

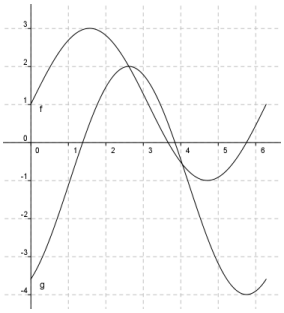
bp: $t = 4$

$N = -60 + 160\sin\frac{\pi}{3{,}4}(t - 4)$

b. bp: $t = 5{,}7$

$N = -60 + 160\cos\frac{\pi}{3,4}(t - 5,7)$

Opgave 39:



a. $f(x) = 1 + 2\sin x$ $\quad$ $g(x) = -1 + 3\sin(x - \frac{1}{3}\pi)$

ev.as: 1 $\quad$ ev.as: -1

amp: 2 $\quad$ amp: 3

per: 2π $\quad$ per: 2π

bp: $(0,1)$ $\quad$ bp: $(\frac{1}{3}\pi, -1)$

b. $y_1 = 1 + 2\sin x$ en $y_2 = -1 + 3\sin(x - \frac{1}{3}\pi)$

intersect geeft: $x = 2,62 \quad \vee \quad x = 4,05$

dus: $0 \le x < 2,62 \quad \vee \quad 4,05 < x \le 2\pi$

c. $y_3 = y_1 + y_2$

optie maximum: $(2,21; 4,36)$

optie minimum: $(5,35; -4,36)$

ev.as: 0

amp: 4,36

per: 2π

bp: $x = 0,64$

$s(x) = 4,36\sin(x - 0,64)$

Opgave 40:

a. maximum: $(0,26; -2,20)$

minimum: $(3,40; -7,80)$

ev.as: $\frac{-7,80 + -2,20}{2} = -5$

amp: $-5 - -7,80 = 2,80$

per: 2π

bp: $x = 0,26$

$s(x) = -5 + 2,80\cos(x - 0,26)$

b. maximum: $(5,78; 0,47)$

minimum: $(2,64; -2,47)$

ev.as: $\frac{0,47 + -2,47}{2} = -1$

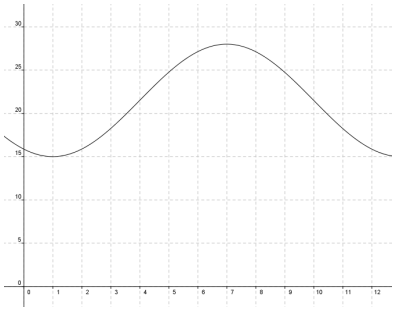
amp: $-1 - -2,47 = 1,47$

per: 2π

bp: $x = 4,21$

$v(x) = -1 + 1,47\sin(x - 4,21)$

Opgave 41:



a. ev.as: 21,5

amp: 6,5

per: $\frac{2\pi}{\frac{1}{6}\pi} = 12$

bp: $(4; 21,5)$

b. $y_1 = 21,5 + 6,5\sin\frac{1}{6}\pi(x - 4)$ en $y_2 = 25$

intersect: $x = 5,086 \quad \vee \quad x = 8,914$

$(8,914 - 5,086) \cdot 30 = 115$ dagen

c. de stijging is het sterkst in het snijpunt met de evenwichts-as, dus op $t = 4$
$\left[\frac{dy}{dx}\right]_{x=4} = 3{,}40° / maand \approx 0{,}8° / week$

d. ev.as: 17,5
amp: $17{,}5 - 15 = 2{,}5$
per: 12 dus $c = \frac{2\pi}{12} = \frac{1}{6}\pi$
bp: $t = 2 + \frac{1}{4} \cdot 12 = 5$
$W = 17{,}5 + 2{,}5 \sin \frac{1}{6}\pi(t - 5)$

8.4 Trilling en trend

Opgave 42:

a. b is de amplitude, dus de maximale uitwijking t.o.v. de evenwichtsstand, dus $b = 25$.
periode: 8 dus $c = \frac{2\pi}{8} = \frac{1}{4}\pi$

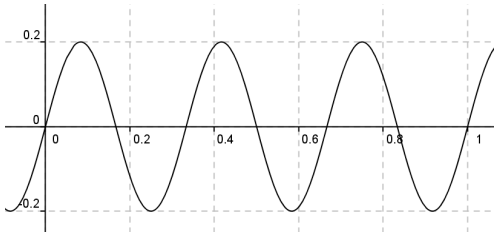
b. $t = 0$: $P'(0,0)$
$t = 2$: $P'(0,25)$

c. $y = 25\sin\frac{1}{4}\pi t$

d. $y = 25\sin(\frac{1}{4}\pi \cdot 6{,}5) = -23{,}1$

Opgave 43:

a. amp: 0,2
per: $\frac{2\pi}{6\pi} = \frac{1}{3}$
freq: $\frac{1}{\frac{1}{3}} = 3$

b. 

Opgave 44:

amp: 3
per: $\frac{2\pi}{60\pi} = \frac{1}{30}$
freq: $\frac{1}{\frac{1}{30}} = 30$

Opgave 45:

$u = 0{,}8\sin(2\pi t) \cdot 440) = 0{,}8\sin 880\pi t$

Opgave 46:

u_1: amp: 3
per: $\frac{100}{3} = 33\frac{1}{3}$ dus $c = \frac{2\pi}{33\frac{1}{3}} = 0{,}06\pi$
$u_1 = 3\sin 0{,}06\pi t$

u_2: amp: 4
per: 40 dus $c = \frac{2\pi}{40} = 0{,}05\pi$
$u_2 = 4\sin 0{,}05\pi t$

Opgave 47:

a. ev.as: 22
amp: 20
per: 75 dus $c = \frac{2\pi}{75} = \frac{2}{75}\pi$
bp: (0,22)
$h = 22 + 20\sin\frac{2}{75}\pi t$

b. $h = 39{,}3$

c. $y_1 = 22 + 20\sin\frac{2}{75}\pi x$ en $y_2 = 32$
intersect geeft: $x = 6{,}25 \quad \vee \quad x = 31{,}25$
dus $31{,}25 - 6{,}25 = 25$ sec

Opgave 48:

a. $\frac{2\pi}{\frac{1}{30}\pi} = 60$ sec

b. $\frac{60}{360} \cdot 60 = 10$ sec

c. $h_B = 22 + 20\sin\frac{1}{30}\pi(t - 10)$

Opgave 49:

a. Q heeft een faseachterstand van $\frac{1}{3}$, dus dat is $\frac{1}{45}$ sec
$x_Q = 20\cos 30\pi(t - \frac{1}{45})$
R heeft een faseachterstand van $\frac{2}{3}$, dus dat is $\frac{2}{45}$ sec
$x_R = 20\cos 30\pi(t - \frac{2}{45})$

b. $x_Q = 20\cos 30\pi(t + \frac{2}{45})$
$x_R = 20\cos 30\pi(t + \frac{1}{45})$

Opgave 50:

a. ev.as: 20
amp: 18
per: 90 dus $c = \frac{2\pi}{90} = \frac{1}{45}\pi$
$h_1 = 20 + 18\sin\frac{1}{45}\pi t$

b. stoeltje 2 heeft een faseachterstand van $\frac{1}{24}$, dus $\frac{1}{24} \cdot 90 = 3{,}75$ sec
$h_2 = 20 + 18\sin\frac{1}{45}\pi(t - 3{,}75)$
stoeltje 22 heeft een fasevoorsprong van $\frac{3}{24}$, dus $\frac{3}{24} \cdot 90 = 11{,}25$ sec
$h_{22} = 20 + 18\sin\frac{1}{45}\pi(t + 11{,}25)$

Opgave 51:

a. per: 50
P en Q: $\frac{12{,}5}{50} = \frac{1}{4}$ dus Q heeft een fasevoorsprong van $\frac{1}{4}$ op P
P en R: $\frac{7{,}5}{50} = \frac{3}{20}$ dus R heeft een faseachterstand van $\frac{3}{20}$ op P
Q en R: $\frac{1}{4} + \frac{3}{20} = \frac{2}{5}$ dus Q heeft een fasevoorsprong van $\frac{2}{5}$ op R

b. per: 50 dus $c = \frac{2\pi}{50} = \frac{1}{25}\pi$
$u_P = 2\sin\frac{1}{25}\pi t$
$u_Q = 2\sin\frac{1}{25}\pi(t + 12{,}5)$
$u_R = 2\sin\frac{1}{25}\pi(t - 7{,}5)$

c. $t = 40$

d. vanaf $t = 0$ tot $t = 5$ dus $\frac{5}{50} \cdot 100\% = 10\%$

Opgave 52:

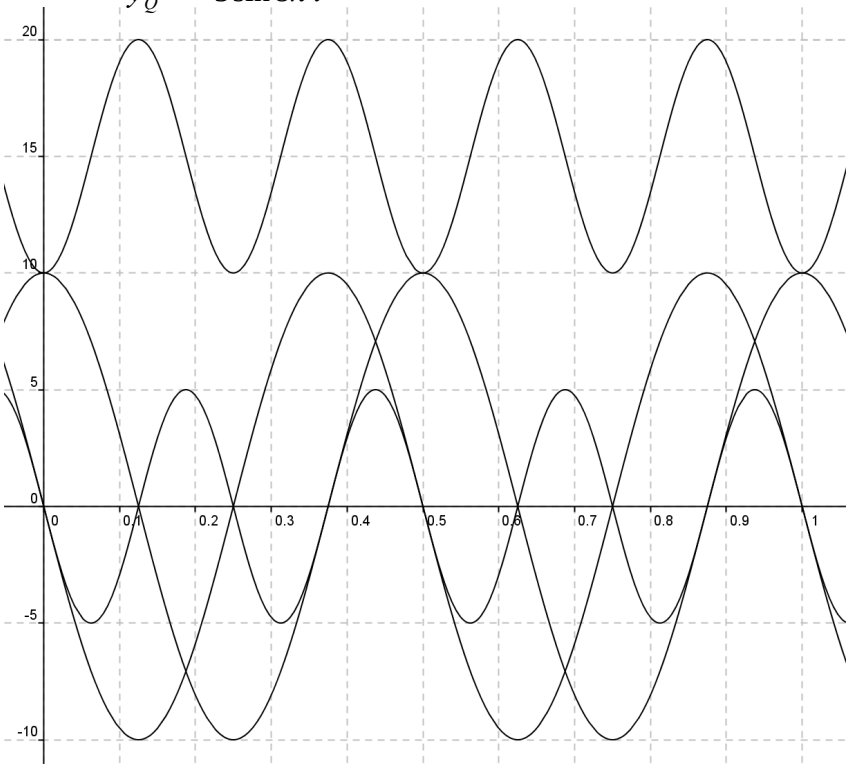
a. de draairichting is tegen de klok in
de straal van rol II is de helft van de straal van rol I

dus de omtrek van rol II is de helft van de omtrek van rol I
dus rol II draait $2\times$ zo snel rond als rol I
dus $freq = 4$ omwentelingen per seconde

b. punt P: amp: 10
per: $\frac{1}{2}$ dus $c = \frac{2\pi}{\frac{1}{2}} = 4\pi$
op $t = 0$ begint de x-coördinaat op zijn hoogste punt en gaat de y-coördinaat dalend door de evenwichtsstand
$x_P = 10\cos 4\pi t$
$y_P = -10\sin 4\pi t$

punt Q: per: $\frac{1}{4}$ dus $c = \frac{2\pi}{\frac{1}{4}} = 8\pi$
op $t = 0$ begint de x-coördinaat op zijn laagste punt en de y-coördinaat gaat dalend door de evenwichtsstand
$x_Q = 15 - 5\cos 8\pi t$
$y_Q = -5\sin 8\pi t$

c.



d. $t = 0 \quad \vee \quad t = \frac{1}{2} \quad \vee \quad t = 1$
de grafieken van x_P en x_Q en die van y_P en y_Q raken elkaar

Opgave 53:

a. 3 januari is van $t = 2$ tot $t = 3$ dus je zoekt het tweede hoogste punt van de grafiek
$y_1 = 55000 - 250x + 1200\sin \pi x$
optie maximum geeft $y = 55578$

b. de hoogste punten dalen in de loop van de tijd

Opgave 54:

a. 708 uur en 44 min = 29,5306 dagen

$\frac{365,2422}{29,5306} = 12,368$ perioden

b. ev.as: $\frac{404336+354340}{2} = 379338$

amp: $404336 - 379338 = 24998$

per: $\frac{29,5306}{365,2422} = 0,08$ jaar dus $c = \frac{2\pi}{0,08} = 77,7$

$A = 379338 + 24998 \sin 77,7t$

c. $d = 379338 + 0,000038t$

d. $387309 - 354340 = 32969$

$\frac{32969}{0,000038} = 8,68 \cdot 10^8 = 868$ miljoen jaar

8.6 Diagnostische toets

Opgave 1:

$A(\cos 40°, \sin 40°) = (0{,}77\,;0{,}64)$

$B(\cos 160°, \sin 160°) = (-0{,}94\,;0{,}34)$

$C(\cos 280°, \sin 280°) = (0{,}17\,;-0{,}98)$

Opgave 2:

a. $\sin\alpha = 0{,}9$

$\alpha = 64°$ geeft de GR

dus $\alpha = 180° - 64° = 116°$

b. $\cos\beta = 0{,}9$

$\beta = 26°$ geeft de GR

dus $\beta = -26°$

$\angle AOB = 116° - -26° = 142°$

Opgave 3:

a. $P(\cos 10, \sin 10) = (-0{,}84\,;-0{,}54)$

b. $P(\cos 5\tfrac{1}{2}\pi, \sin 5\tfrac{1}{2}\pi) = (0,-1)$

Opgave 4:

a. $\tfrac{3}{4}\pi \ rad = \tfrac{3}{4}\cdot 180 = 135°$

b. $\tfrac{1}{5}\pi \ rad = \tfrac{1}{5}\cdot 180 = 36°$

c. $0{,}6 \ rad = \tfrac{0{,}6}{\pi}\cdot 180 = 34{,}4°$

d. $26\pi \ rad = 26\cdot 180 = 4680°$

e. $\tfrac{2}{3}\pi \ rad = \tfrac{2}{3}\cdot 180 = 120°$

f. $\tfrac{2}{3} \ rad = \tfrac{\frac{2}{3}}{\pi}\cdot 180 = 38{,}2°$

Opgave 5:

a. $270° = \tfrac{270}{180}\cdot\pi = 1\tfrac{1}{2}\pi \ rad$

b. $-60° = \tfrac{-60}{180}\cdot\pi = -\tfrac{1}{3}\pi \ rad$

c. $150° = \tfrac{150}{180}\cdot\pi = \tfrac{5}{6}\pi \ rad$

d. $330° = \tfrac{330}{180}\cdot\pi = 1\tfrac{5}{6}\pi \ rad$

e. $40° = \tfrac{40}{180}\cdot\pi = \tfrac{2}{9}\pi \ rad$

f. $-70° = \tfrac{-70}{180}\cdot\pi = -\tfrac{7}{18}\pi \ rad$

Opgave 6:

a. $\alpha = 0{,}82$

b. $\alpha = 0{,}54$

c. $\alpha = 0{,}79$

Opgave 7:

a. $y = \sin x \xrightarrow{V_{x\text{-as},3}} y = 3\sin x \xrightarrow{T(\frac{1}{2}\pi,2)} y = 2 + 3\sin(x - \tfrac{1}{2}\pi)$

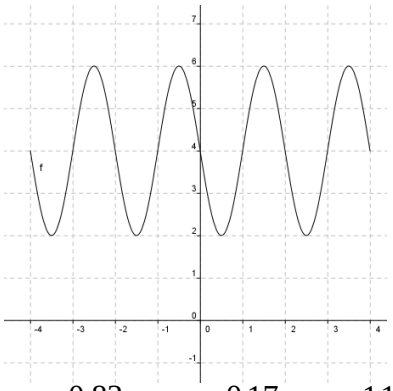
b. $y = \sin x \xrightarrow{V_{y-as,\frac{1}{2}}} y = \sin 2x \xrightarrow{T(\frac{1}{4},-3)} y = -3 + \sin 2(x - \frac{1}{4})$

Opgave 8:

$y = \cos x \xrightarrow{T(\frac{3}{4}\pi,1)} y = 1 + \cos(x - \frac{3}{4}\pi) \xrightarrow{V_{y-as,4}} y = 1 + \cos(\frac{1}{4}x - \frac{3}{4}\pi) \xrightarrow{V_{x-as,2}}$

$y = 2 - 2\cos(\frac{1}{4}x - \frac{3}{4}\pi)$

Opgave 9:



a. ev.as: 4
amp: 2
per: $\frac{2\pi}{\pi} = 2$
bp: (1,4)

b. $y_1 = 4 + 2\sin\pi(x-1)$
$y_2 = 5$
intersect geeft:
$x = -2{,}83 \vee x = -2{,}17 \vee x = -0{,}83 \vee x = -0{,}17 \vee x = 1{,}17 \vee x = 1{,}83 \vee x = 3{,}17 \vee x = 3{,}83$
$-2{,}83 < x < -2{,}17 \vee -0{,}83 < x < -0{,}17 \vee 1{,}17 < x < 1{,}83 \vee 3{,}17 < x < 3{,}83$

c. $\left[\frac{dy}{dx}\right]_{x=1} = 6{,}28$

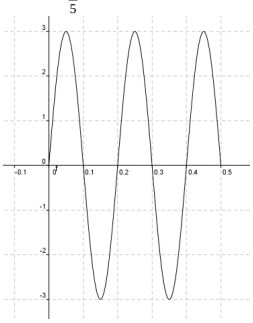
Opgave 10:

a. ev.as: -10
amp: 20
per: 30 dus $c = \frac{2\pi}{30} = \frac{1}{15}\pi$
bp: (10,−10)
$f(x) = -10 + 20\sin\frac{1}{15}\pi(x-10)$

b. bp: $x = 17{,}5$
$f(x) = -10 + 20\cos\frac{1}{15}\pi(x-17{,}5)$

Opgave 11:

a. amp: 3
trillingstijd: $\frac{2\pi}{10\pi} = \frac{1}{5}$
freq: $\frac{1}{\frac{1}{5}} = 5$

b. 

c. $\frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$
$u_Q = 3\sin 10\pi(t - \frac{1}{25})$

Opgave 12:

a. $AB = \sqrt{100^2 + 21^2} = 102{,}2$

$omtrek = 2\pi \cdot r = 2\pi \cdot 0{,}25 = 0{,}5\pi = 1{,}57$

$\frac{102{,}2}{1{,}57} = 65$ omwentelingen

b. per: 2 sec

dus $65 \cdot 2 = 130$ sec

c. $\frac{102{,}2}{130} = 0{,}786\,{}^{m}\!/_{s} = 2{,}83\,{}^{km}\!/_{uur}$